Learning Parameterized Families of Games

Madelyn Gatchel and Bryce Wiedenbeck

COMPUTER SCIENCE AND ENGINEERING
UNIVERSITY OF MICHIGAN

DAVIDSON

Outline

What is a parameterized family of games?
1

Where is there a need for parameterized game families?
2

How do we learn a parameterized game model?
3

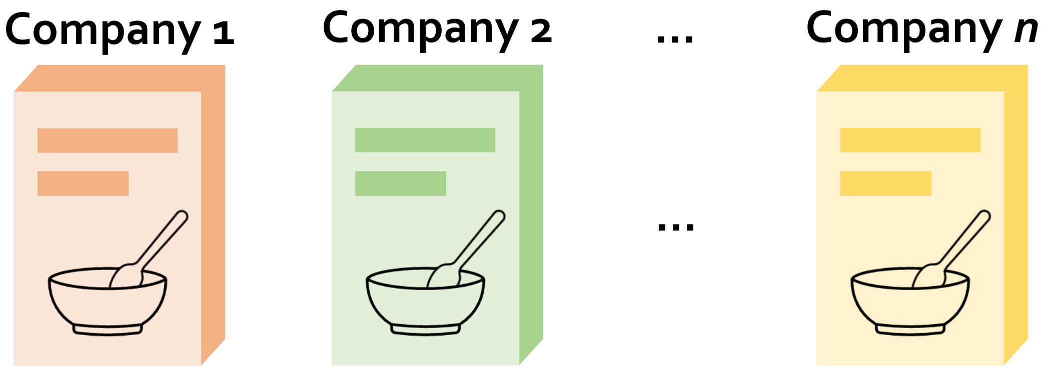
How do we analyze a parameterized game model?
4

Motivating Example: Launching New Cereal

Company 1 **Company 2** … **Company *n***

- **Parameter $\boldsymbol{V}$:** $\Pr(Recession)$
  - Must be fixed in an NFG

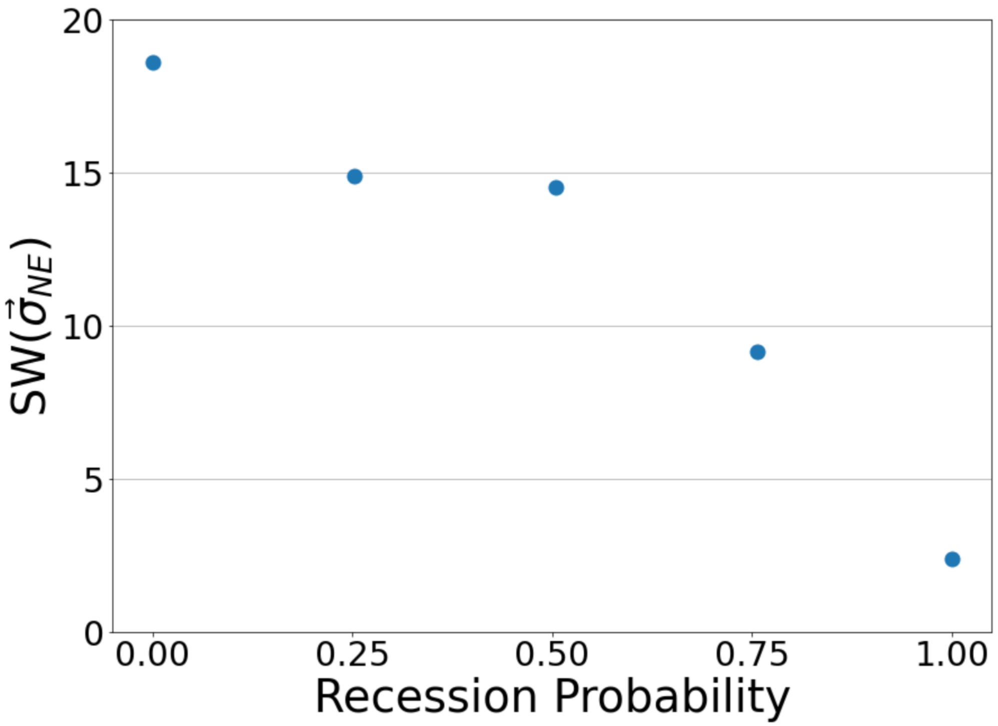
Motivating Example: Launching New Cereal



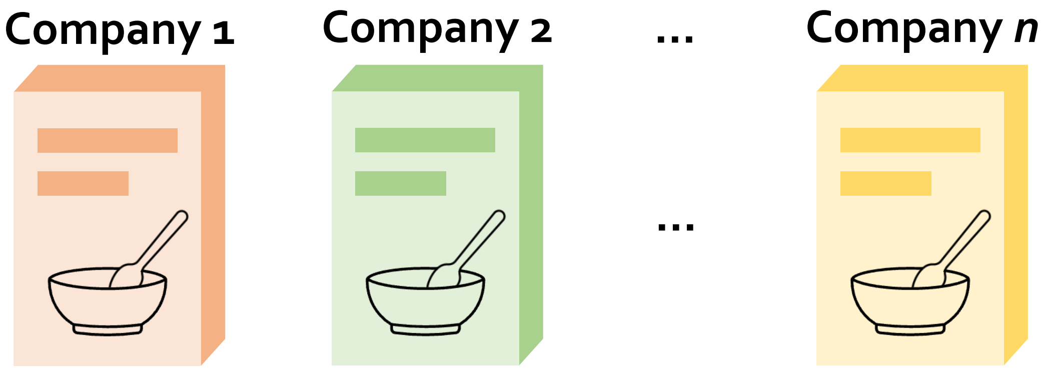
- **Parameter *V*:** $\Pr(Recession)$
  - Must be fixed in an NFG

Separate Game Instances:

$\Gamma(0); \Gamma(.25); \Gamma(.5); \Gamma(.75); \Gamma(1)$



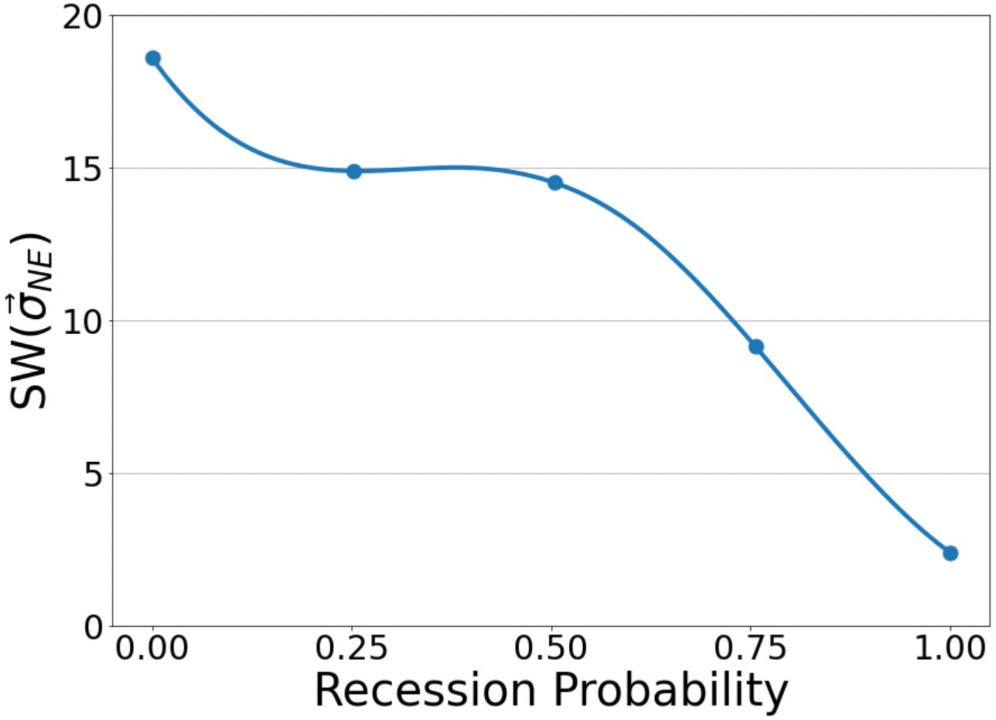
Motivating Example: Launching New Cereal



- **Parameter *V*:** $\Pr(Recession)$
  - Must be fixed in an NFG

Parameterized Game Family:

$$\gamma(V) = \{\Gamma(v) : 0 \leq v \leq 1, v \in \mathbb{R}\}$$



Hypothesis

Games which are related by a common, ordinal environment parameter likely have related payoff functions.

Outline

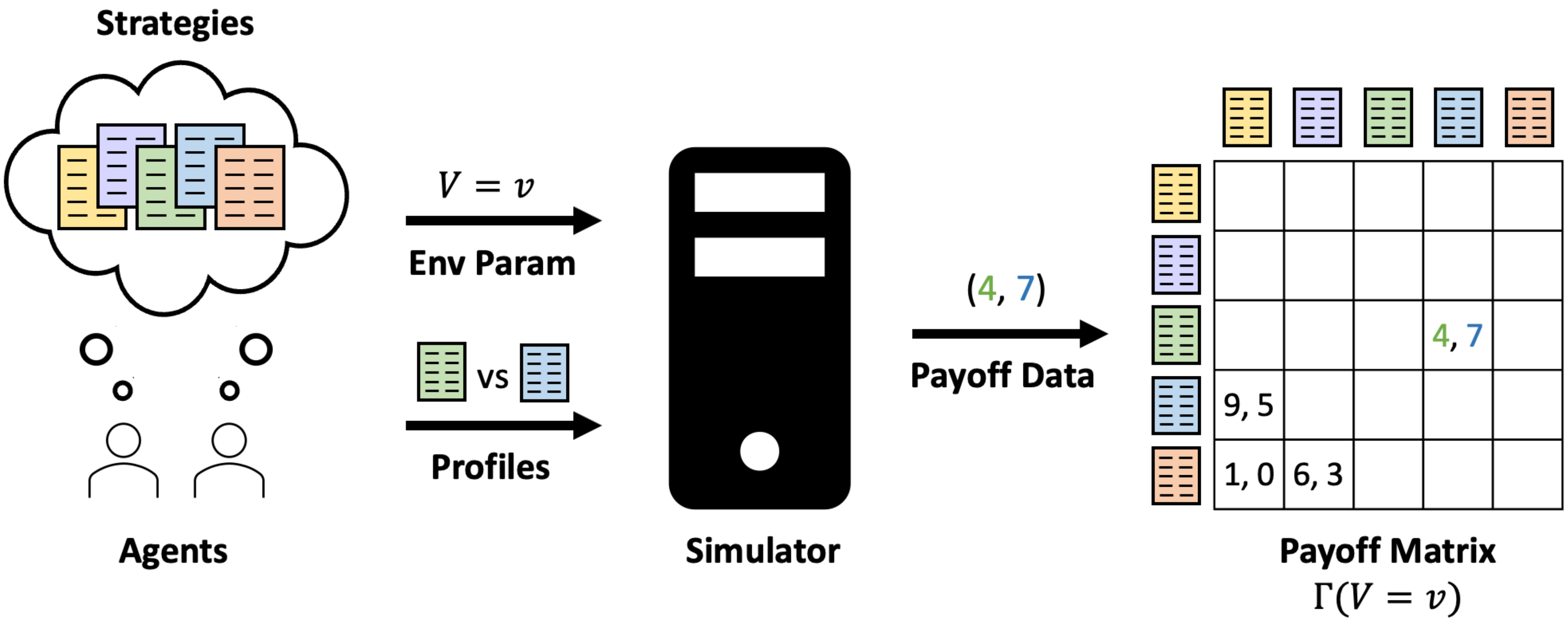
What is a parameterized family of games? 1

Where is there a need for parameterized game families? 2

How do we learn a parameterized game model? 3

How do we analyze a parameterized game model? 4

Simulation-Based Games



Example SBGs

Discrete Parameters

- Latency arbitrage in financial markets [21]
  - Number of background traders
- Complex network routing protocols [24]
  - Number of non-attacking nodes
    - (Clients, ISPs, roots, and servers)

Continuous Parameters

- Debt cycle compression among financial institutions [14]
  - Asset recovery rate
- Strategic market selection with fast and slow traders [20]
  - Mean reversion parameter

Outline

What is a parameterized family of games?

Where is there a need for parameterized ame families?

2

Game Theory Background

2.5

How do we lea parameterized game model?

3

do we analyze a parameterized game model?

4

Background

- In a **symmetric game**, player identities are irrelevant
- **Opponent profile:** $\vec{s}$
- $u_j(\vec{s})$ denotes payoff for players playing strategy j

Background

- In a **symmetric game**, player identities are irrelevant
- **Opponent profile:** $\vec{s}$
- $u_j(\vec{s})$ denotes payoff for players playing strategy j
- **Symmetric profile:** $\vec{\sigma}$
- **Deviation payoff:**

$$u_j(\vec{\sigma}) = \sum_{\vec{s} \in \vec{S}} \Pr(\vec{s} \mid \vec{\sigma})\, u_j(\vec{s})$$

- **Deviation payoff vector:** $u(\vec{\sigma})$
- **Expected utility:** $\vec{\sigma} \cdot u(\vec{\sigma})$

Background

- **Regret:**

$$\epsilon(\vec{\sigma}) = \underbrace{\max_{j \in S} u_j(\vec{\sigma})}_{\text{max deviation payoff}} - \underbrace{\vec{\sigma} \cdot u(\vec{\sigma})}_{\text{expected utility}}$$

- **Nash equilibrium:**

$\vec{\sigma}$ such that $\varepsilon(\vec{\sigma}) = 0$

- **ε-Nash equilibrium:**

$\vec{\sigma}$ such that $\epsilon(\vec{\sigma}) \leq \varepsilon$

Background

- **Regret:**

$$\epsilon(\vec{\sigma}) = \underbrace{\max_{j \in S} u_j(\vec{\sigma})}_{\textbf{max deviation payoff}} - \underbrace{\vec{\sigma} \cdot u(\vec{\sigma})}_{\textbf{expected utility}}$$

- **Nash equilibrium:**

  $\vec{\sigma}$ such that $\varepsilon(\vec{\sigma}) = 0$

- **ε-Nash equilibrium:**

  $\vec{\sigma}$ such that $\epsilon(\vec{\sigma}) \leq \varepsilon$

Focus of this work

Outline

What is a parameterized family of games? 1

Where is there a need for parameterized game families? 2

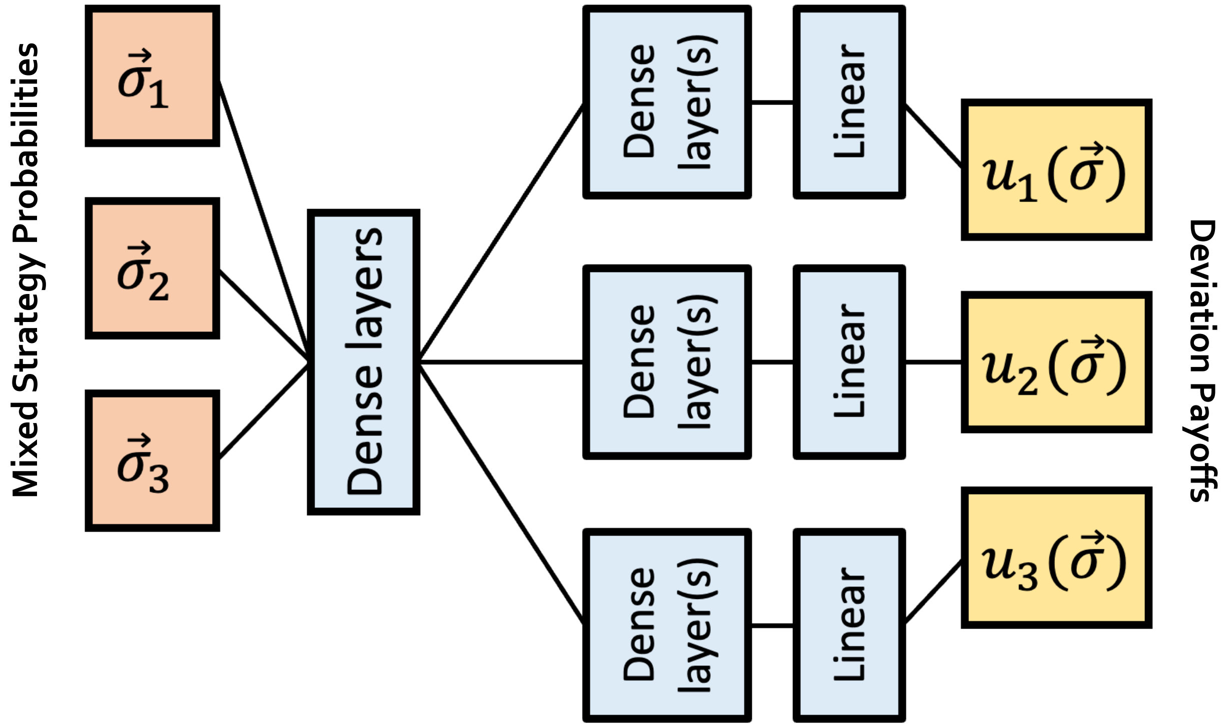
How do we learn a parameterized game model? 3

How do we analyze a parameterized game model? 4

Learning Deviation Payoffs in Simulation-Based Games

- Sokota et al. [15] use a **multi-headed neural network** to learn the deviation payoff function

- This learned function may be used in approximate Nash-finding algorithms to find ϵ-Nash equilibria

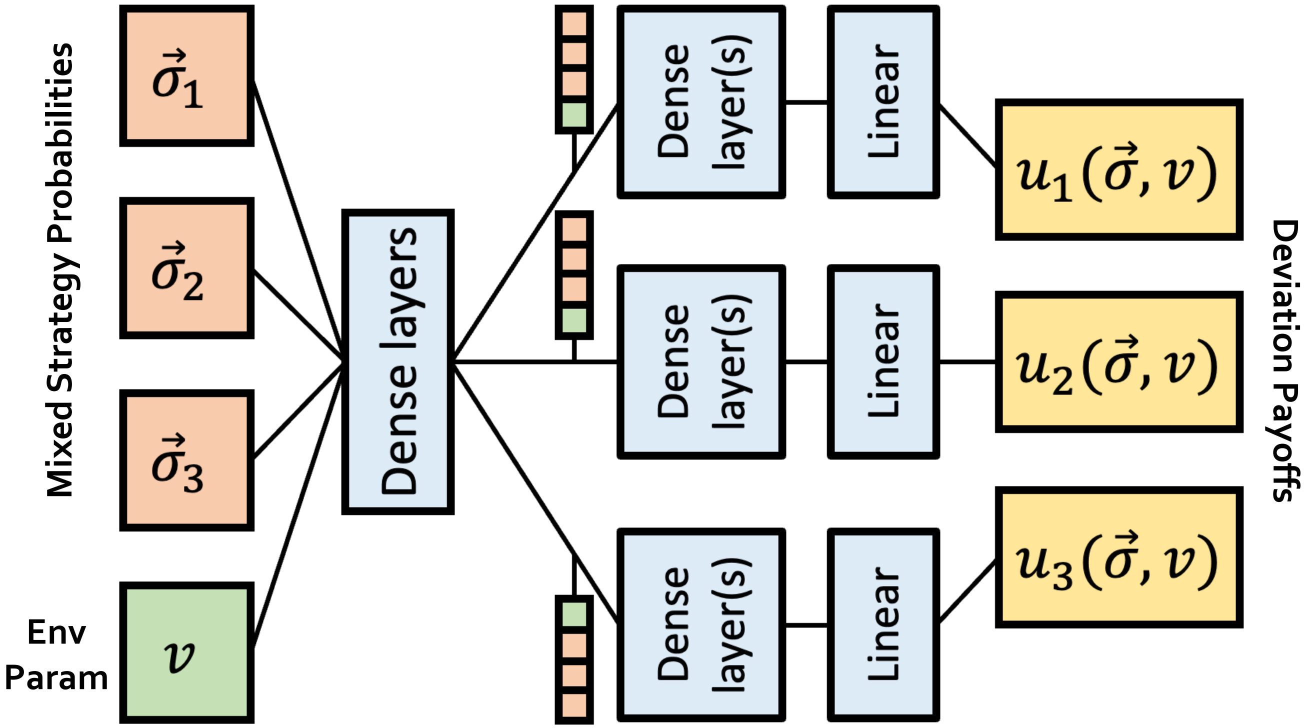


Neural network architecture for a 3-strategy symmetric game instance

Learning Deviation Payoffs in Parameterized Game Families

- We adapt [15] to learn a mapping from mixed-strategy profiles ***and environment parameters*** to deviation payoffs

- This network has a **skip connection** from the input layer to each strategy head



Neural network architecture for a 3-strategy symmetric parameterized game family

Experiment 1a: Comparison to Existing Work

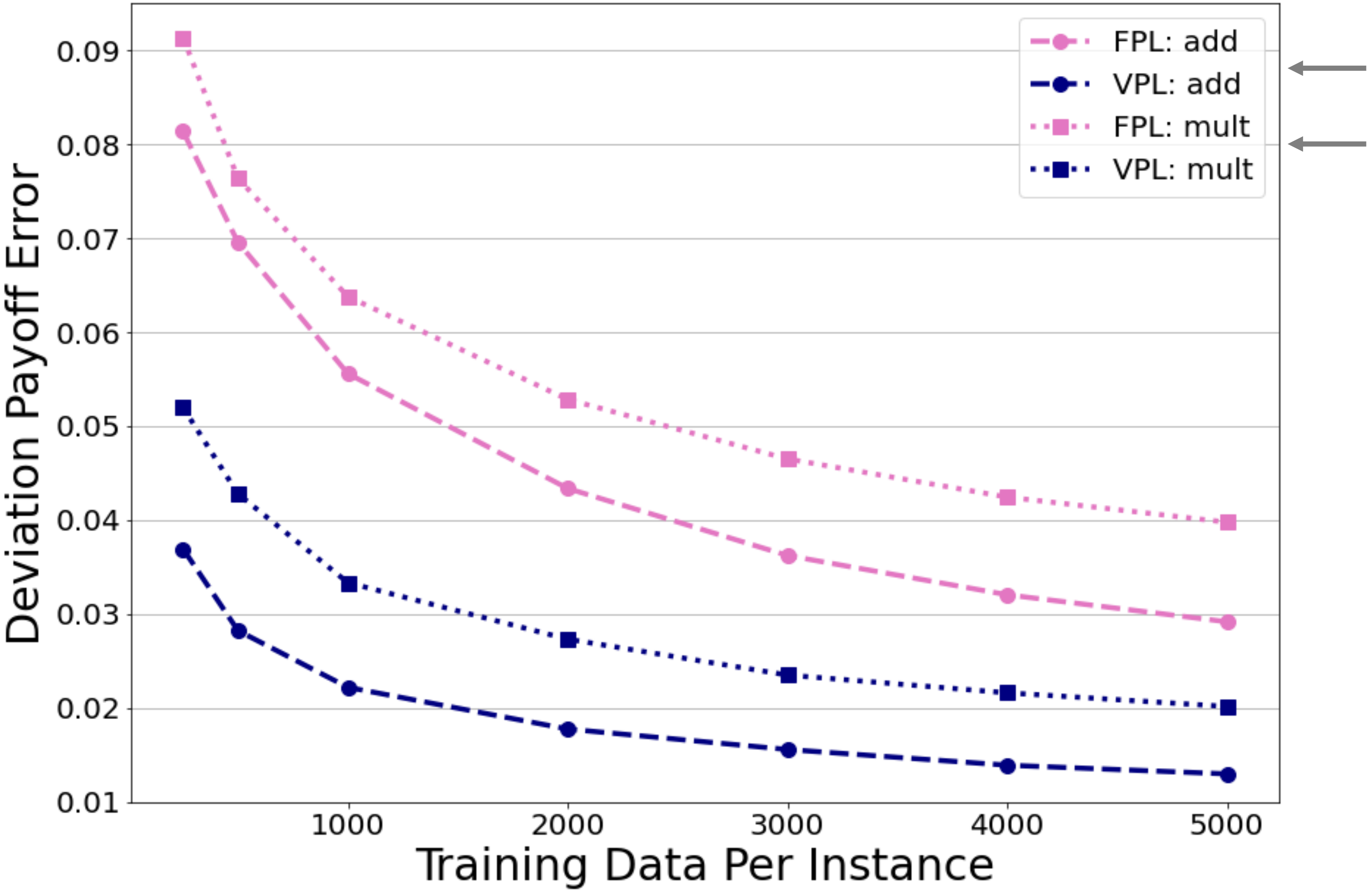
Varied Parameter: Number of Players [90-100]

FPL

Fixed-Parameter Learning [15]

VPL

Variable-Parameter Learning [this work]



Random game classes used in prior literature

Experiment 1b: Comparison to Existing Work

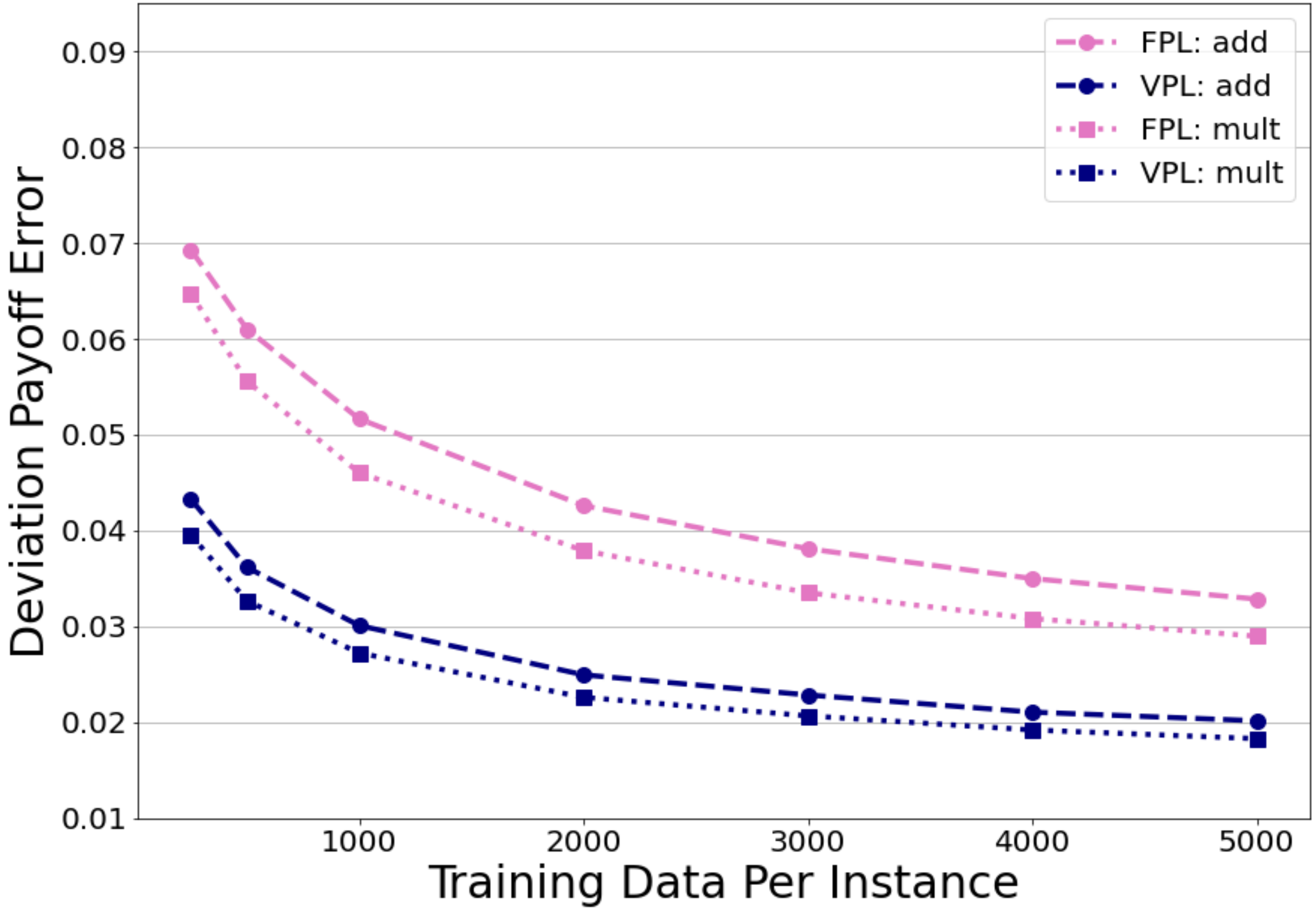
Varied Parameter: Erdős–Rényi Probability Threshold [0.15-0.25]

FPL

Fixed-Parameter Learning [15]

VPL

Variable-Parameter Learning [this work]



Random game classes used in prior literature

Outline

What is a parameterized family of games?
1

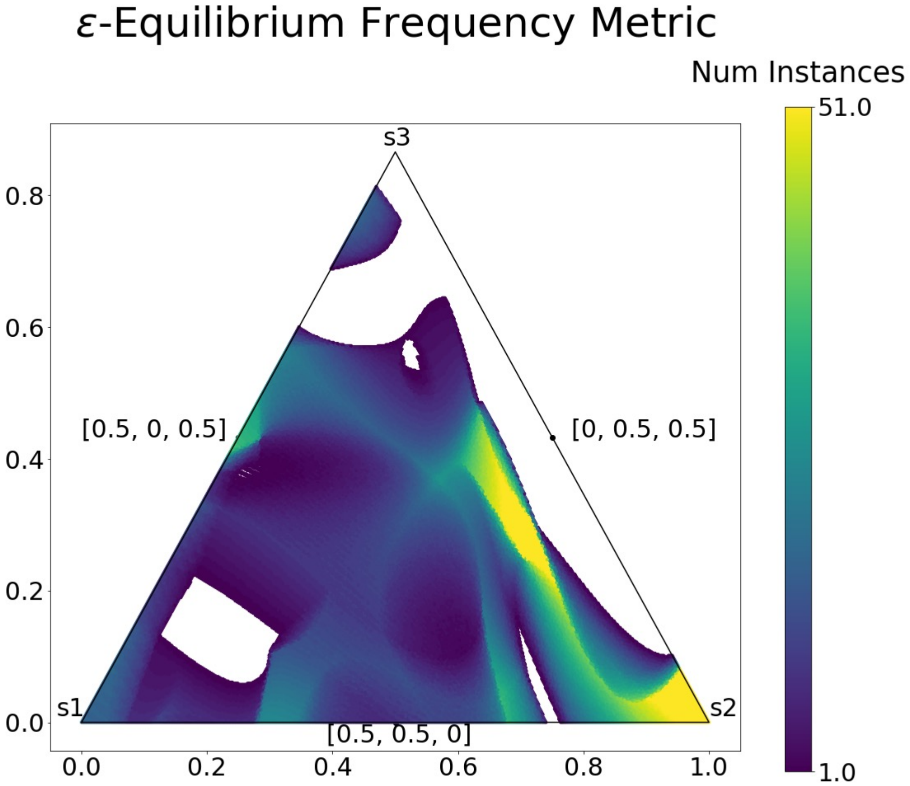
Where is there a need for parameterized game families?
2

How do we learn a parameterized game model?
3

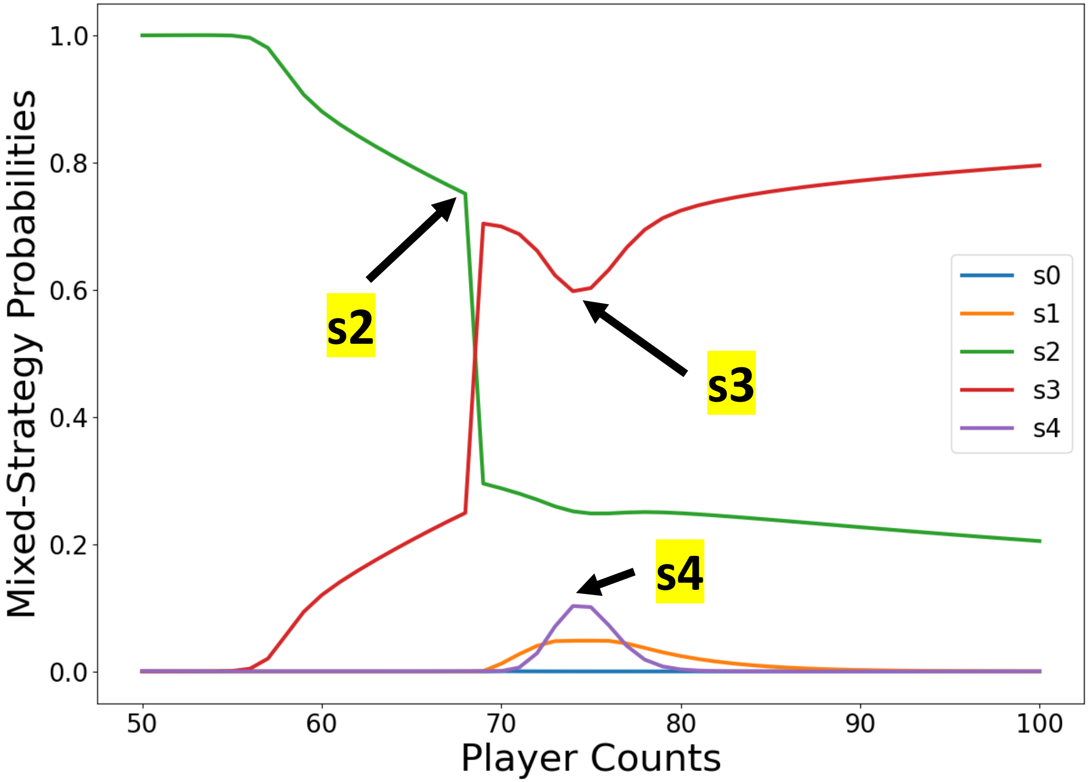
How do we analyze a parameterized game model?
4

Parameterized Game Family Analysis

Identifying Robust Equilibria



Parameter Sensitivity Analysis



By learning a single model for a parameterized game family, we can:

1. Achieve higher payoff accuracy with less data

2. Conduct new types of robustness and sensitivity analyses

3. Better characterize parameter impact on strategic incentives

References

[14] Katherine Mayo and Michael P. Wellman. 2021. A Strategic Analysis of Portfolio Compression. In *ICAIF*. Article 20, 8 pages.

[15] Sam Sokota, Caleb Ho, and Bryce Wiedenbeck. 2019. Learning Deviation Payoffs in Simulation -Based Games. In *AAAI*, Vol. 33. 2173-2180.

[20] Elaine Wah, Dylan Hurd, and Michael P. Wellman. 2016. Strategic Market Choice: Frequent Call Markets vs. Continuous Double Auctions for Fast and Slow Traders. *EAI Endorsed Transactions on Serious Games* 3, 10 (2016).

[21] Elaine Wah and Michael P. Wellman. 2016. Latency Arbitrage in Fragmented Financial Markets: A Strategic Agent-Based Analysis. *Algorithmic Finance* 5, 304, 69-93.

[24] Michael P. Wellman, Taie Hyung Kim, and Quang Duong. 2013. Analyzing Incentives for Protocol Compliance in Complex Domains: A Case Study of Introduction-Based Routing. In *Workshop on the Economics of Information Security.*

Thank you!

Madelyn Gatchel
University of Michigan
gatchel@umich.edu

Link to Paper:
Learning Parameterized Families of Games

Bryce Wiedenbeck
Davidson College
brwiedenbeck@davidson.edu