Analyzing Games with a Variable Number of Players

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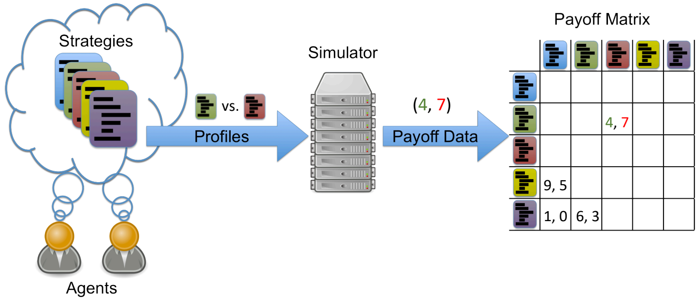
Background

- **Normal-form game:** a mathematical model of incentives which includes
  - A fixed set of players
  - The possible actions they can play
  - Each player's payoff function
- **Regret:** the maximum payoff amount any player can gain by deviating to any other strategy
- **Nash equilibrium:** a set of strategies such that no player has an incentive to deviate

	R	P	S
R	0, 0	-1, 1	1, -1
P	1, -1	0, 0	-1, 1
S	-1, 1	1, -1	0, 0

Empirical Game Theory

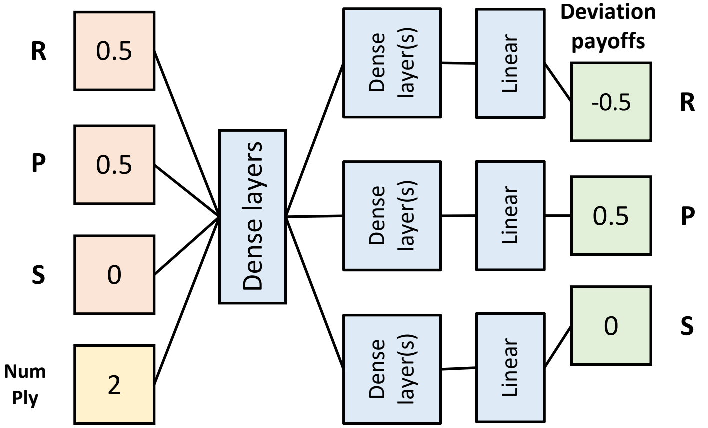
Our techniques are particularly relevant for analyzing **simulation-based games**, where the payoff matrix is not known in advance but can be filled through a series of multi-agent simulations.



Because the interaction we are studying might have an uncertain number of participants, a game with a fixed number of players might be an insufficient model.

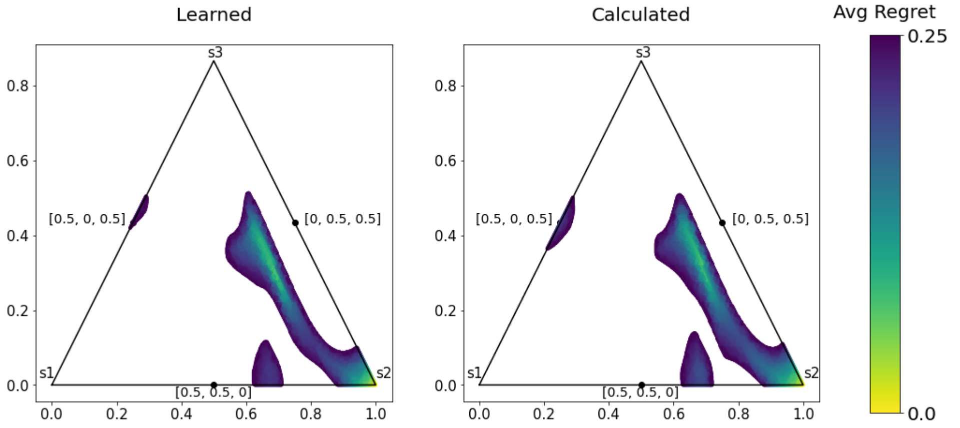
Deviation Payoff Learning Model

- **Hypothesis:** the payoffs in a game with x players are similar or related to the payoffs in the same game with $x \pm 1$ players, given a large value of x
- We generalize prior work to analyze **games with a large, variable number of players**, where the number of players falls in a specified range
- We use a **multi-headed neural network** to learn a mapping from mixed strategy profiles and number of players to deviation payoffs
- **Deviation payoff:** the expected payoff a player would receive by deviating or changing strategies, given the mixed strategies everyone else is playing



Equilibrium Robustness Metrics

Average Regret. The regret of a candidate Nash equilibrium is computed for each instance of the game and then averaged. If the average regret falls under some specified regret threshold, we say the candidate equilibrium is robust.



Approximate Equilibrium Frequency. If the number of instances in which the candidate Nash equilibrium is an approximate equilibrium (for a fixed ϵ) is higher than some threshold, we say the candidate equilibrium is robust.